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Strict superharmonicity of Mityuk's function for countably connected domains of simple structure

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Abstract

© 2017, Pleiades Publishing, Ltd.Strict superharmonicity of generalized reduced module as a function of a point (we call it Mityuk's function) is established for the subclass of countably connected domains with unique limit point boundary component. The function just mentioned was first studied in detail by I.P. Mityuk and plays now an important role in the research of the exterior inverse boundary value problems of the theory of analytic functions in the multiply connected domains. At the heart of such a research one can see the fact that the critical points of Mityuk's function are only maxima, saddles or semisaddles of corresponding surface. This fact is followed from the above strict superharmonicity.

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Keywords

countably connected domain, exterior inverse boundary value problem, Gakhov equation, Generalized reduced module, inner mapping (conformal) radius, Mityuk's function, Mityuk's radius

References

[1] M. T. Nuzhin, "On some inverse boundary value problems and their applications to the definition of sectional shape of twisted rods," Uch. Zap. Kazan. Univ. 109 (1), 97–120 (1949).

[2] F. D. Gakhov, "On the inverse boundary problems," Dokl. Akad. Nauk SSSR 86 (4), 649–652 (1952).

[3] L. A. Aksent'ev, M. I. Kinder, and S. B. Sagitova, "Solvability of the exterior inverse boundary value problem in the case of multiply connected domain," Tr. Semin. Kraev. Zadacham 20, 22–34 (1983).

[4] M. I. Kinder, "The number of solutions of F. D. Gakhov's equation in the case of a multiply connected domain," Izv. Vyssh. Uchebn. Zaved., Mat. 28 (8), 69–72 (1984).

[5] M. I. Kinder, "Investigation of F. D.Gakhov's equation in the case of multiply connected domains," Tr. Semin. Kraev. Zadacham 22, 104–116 (1985).

[6] L. A. Aksent'ev, A. M. Elizarov, and M. I. Kinder, "Inverse boundary value problems for multiply connected domains on Riemann surfaces of genus zero," Tr. Semin. Kraev. Zadacham 21, 19–32 (1984); Tr. Semin. Kraev. Zadacham 22, 16–29 (1985); Tr. Semin. Kraev. Zadacham 23, 25–36 (1987).

[7] L. A. Aksent'ev, A. M. Elizarov, and M. I. Kinder, "Continuation of F. D. Gakhov's work in inverse boundary value problems," in Proceedings of the Commemorative Seminar on Boundary Value Problems Dedicated to the 75th Birthday of Acad. F. D. Gakhov (Minsk, 1981), pp. 139–142.

[8] M. I. Kinder, "Exterior inverse boundary value problem in multiply connected regions and on Riemann surfaces," Cand. Sci. (Math.) Dissertation (Kazan State Univ., Kazan, 1984).

[9] A. V. Kazantsev, "Extremal properties of the inner radius and their applications," Cand. Sci. (Math.) Dissertation (Kazan State Univ., Kazan, 1990).

[10] L. A. Aksent'ev, A. V. Kazantsev, M. I. Kinder, and A. V. Kiselev, "Classes of uniqueness of an exterior inverse boundary value problem," Tr. Semin. Kraev. Zadacham 24, 39–62 (1990).

[11] L. A. Aksent'ev, A. V. Kazantsev, and M. I. Kinder, "On classes of uniqueness of an exterior inverse boundary value problem," in Proceedings of the 2nd Mathematical Readings in Memory of M. Ya. Suslin, Sept. 23–28, Saratov, 1991, p. 61.

[12] A. V. Kiselev, "Geometric properties of solutions of the exterior inverse boundary value problem," Izv. Vyssh. Uchebn. Zaved., Mat. 36 (7), 20–25 (1992).

[13] A. V. Kazantsev and M. I. Kinder, "Solvability of the exterior inverse boundary value problem in the case of multiply connected domains," in Proceedings of the 11th International Conference on Algebra and Analysis, Dedicated to the 100th Anniversary of the Birth of N. G. Chebotarev, Kazan, June 5–11, 1994, Vol. 2, pp. 65–66.

[14] I. P. Mityuk, "A generalized reduced module and some of its applications," Izv. Vyssh. Uchebn. Zaved., Mat., No. 2, 110–119 (1964).

[15] A. V. Kazantsev, "Gakhov set in the Hornich space under the Bloch restriction on pre-Schwarzians," Uch. Zap. Kazan. Univ., Ser. Fiz.-Mat. Nauki 155 (2), 65–82 (2013).

[16] A. V. Kazantsev, "Zmorovich's method in the problem of investigation of Mityuk's functional," in Proceedings of the 7th International Symposium on Fourier Series and Their Applications, Rostov, 2014, pp. 79–80.

[17] H. Grötzsch, "Über konforme Abbildung unendlich vielfach zusammenhängender schlichter Bereiche mit endlich vielen Häufungsrandkomponenten," Ber. Sächs. Akad. Wiss. Leipzig, Math.-Phys. Kl. 81, 51–86 (1929).

[18] H. Grötzsch, "Das Kreisbogenschlitztheorem der konformen Abbildung schlichter Bereiche," Ber. Sächs. Akad.Wiss. Leipzig, Math.-Phys. Kl. 83, 238–253 (1931).

[19] E. Reich and S. E. Warschawski, "On canonical conformal maps of regions of arbitrary connectivity," Pacif. J. Math. 10 (3), 965–989 (1960).

[20] S. Bergman and M. Schiffer, "Kernel functions and conformalmapping," Compos. Math. 8, 205–249 (1951).

[21] G. M. Golusin, Geometric Theory of Functions of a Complex Variable, Vol. 26 of Transl. Math. Monographs (Am.Math. Soc., Providence, 1969).

[22] M. Schiffer, "Hadamard's formula and variation of domain-functions," Am. J. Math. 68 (4), 417–448 (1946).

[23] W. Hayman and P. Kennedy, Subharmonic Functions, (Academic, London, 1976), Vol.1.

[24] S. Stoilov, The Theory of Functions of a Complex Variable (Fizmatgiz, Moscow, 1962), Vol. 2.